BLACK HOLES, THE WHEELER-DE WITT EQUATION
AND THE SEMI-CLASSICAL APPROXIMATION

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ABSTRACT

The definition of matter states on spacelike hypersurfaces of a 1+1 dimensional black hole spacetime is considered. Because of small quantum fluctuations in the mass of the black hole, the usual approximation of treating the gravitational field as a classical background on which matter is quantized, breaks down near the black hole horizon. On any hypersurface that captures both infalling matter near the horizon and Hawking radiation, a semiclassical calculation is inconsistent. An estimate of the size of correlations between the matter and gravity states shows that they are so strong that a fluctuation in the black hole mass of order $e^{-M/M_{Planck}}$ produces a macroscopic change in the matter state. (Based on a talk given at the 7th Marcel Grossmann Meeting on work in collaboration with E. Keski-Vakkuri, G. Lifschytz and S. Mathur.)

Various authors have recently argued that the information paradox in black hole evaporation could be resolved if there is a mechanism that allows the information falling into a black hole to be transferred to the outgoing radiation at the black hole horizon. With the additional assumption that an infalling observer sees nothing unusual occuring at the horizon, it follows that observations of the Hawking radiation at future null infinity and of infalling matter at the black hole horizon must be complementary in some sense[1]. This has been expressed by 't Hooft[2] as the statement that operators corresponding to each observation must be strongly non-commuting despite the fact that the two regions of interest, the horizon and future null infinity, are spacelike separated in the usual background geometry of an evaporating black hole. Calculations in 1+1 dimensions seem to support this point of view[3], although assumptions are needed about a boundary condition at the region of strong coupling.

The usual derivation of the black hole paradox assumes a picture of quantum fields in a classical background spacetime. This assumption can be justified by appealing to a foliation of the black hole spacetime which avoids all regions of strong curvature (or strong coupling) and which includes late time hypersurfaces which capture the outgoing radiation and the infalling matter close to the horizon. A hypersurface of this last kind will be referred to as an S-surface (the potential importance of these kinds of surfaces has been emphasized by Susskind[1]).

To ask whether a semi-classical description of black hole physics is trustworthy, it is necessary to consider a quantum theory of gravity and matter, and to understand how the semi-classical approximation is obtained[4]. The Wheeler-DeWitt equation can be expanded in the Planck mass, leading to a natural separation between a quasi-classical gravitational state and a quantum matter state satisfying the functional Schrödinger equation in the gravitational background. This separation bewteen classical and quantum variables is thought to be reliable whenever the inherenet scales in a problem are small compared to the Planck scale. For example, close to a black hole horizon, where part of an S-surface is located, the energy density is relatively small, as are all co-ordinate invariant local quantities. However, it was suggested in Ref. 5 that the copious particle production associated with the black hole evaporation process might lead to excessive decoherence of the gravitational field, spoiling

it's classical behaviour. It will be explained below that a careful analysis of the semi-classical approx-imation shows that other scales in the black hole problem, not just the local invariants, can spoil the expansion. It turns out to fail precisely when one tries to obtain the state of matter on an S-surface. Below I shall briefly summarize the key results of Ref. 6, where a detailed calculation can be found that shows the breakdown of the semi-classical approximation in the 1+1 dimensional CGHS model.

Working in 1+1 dimensions, the configuration space of the gravitational field is the space of 1-geometries. A 1-geometry is an equivalence class of metrics under the action of spatial diffeomor-phisms. In the model that we consider in Ref. 6, where a dilaton is present as part of the gravitational field, a 1-geometry is defined by the function $\phi(s)$ giving the dependence of the dilaton field on the proper distance along a hypersurface. This must be supplemented by a boundary condition giving the point of origin of the proper distance.

If we include a matter field, we can express the configuration space of matter fields as the space of functions $f(s)$. A solution to the Wheeler-DeWitt equation including matter then takes the form

$$\Psi[\phi(s), f(s)] \tag{1}$$

We wish to approximate this functional by a solution

$$\psi_{\mathcal{M}}[f(s), \phi(s)] \tag{2}$$

of the functional Schrödinger equation on a mean background spacetime $\mathcal{M}$ times a quasi-classical gravitational state approximating $\mathcal{M}$ and depending only on $\phi(s)$. Note that given a spacetime $\mathcal{M}$, the function $\phi(s)$ (plus an appropriate boundary condition) specifies a unique timeslice in $\mathcal{M}$, and so acts like the time variable for the Schrödinger picture state, just as s acts like a spatial coordinate; the boundary condition for an asymptotically flat spacetime is just the location of the hypersurface at spacelike infinity which itself defines the zero-point of the proper distance.

For the black hole, we make a separation between the matter forming the black hole, which we include as part of the quasi-classical variables, and the fields that give rise to the Hawking radiation which are denoted by the field f. Regardless of how the black hole is formed, the quasi-classical state representing the classical variables cannot be exactly classical. For example, the infalling matter cannot be in an exact energy eigenstate. If it is to be sufficiently localised to form a black hole, there must be a spread in its energy of at least $1/M$ where M is the mass of the black hole in Planck units. Thus the background spacetime has a mass that is uncertain by at least $1/M$. When we write the Schrödinger equation, it is important to remember these Planck scale uncertainties in the mean background spacetime. If the semi-classical approximation is reliable, the uncertainties should not affect matter propagation. There is a simple criterion for determining whether or not this is the case. Consider two different choices for the mean background spacetime which are solutions with masses that differ on the order of $1/M$ or below. A priori each solution can be taken as a mean background for the semi-classical approximation. If we are to write the state of gravity and matter as the product of a gravitational state, which contains both mean spacetimes in its spread, and a single Schrödinger type matter state, then it must make no difference to the matter state which of the two mean spacetimes we take to be the background.

Let us make this statement more precise. We are interested in obtaining a functional giving the state of matter on 1-geometries. Consider the two mean spacetimes described above and any 1-geometry that embeds in both. Suppose that we are given a natural way of comparing the matter states (defined using quantum field theory) on each of the spacetimes, through the common 1-geometry. Then the criterion for semi-classicality is that the two matter states be virtually indistinguishable when compared on any choice of common 1-geometry. Suppose we take as an intial condition that the two states be indistinguishable at past null infinity. There is no guarantee that this will continue to be true

under dynamical evolution of the states in the respective spacetimes. In all familiar physical situations the states do remain indistinguishable, meaning that quantum gravity effects can be ignored. However, a black hole is different: by the time states are evolved to the type of common 1-geometry which we have called an S-surface, the states become radically different. This result shows that the quantum state of a black hole cannot be approximated as a quasi-classical gravitational state times a single matter state defined using qauntum field theory on the gravitational background.

In Ref. 6 details are given of how to indentify 1-geometries in different spacetimes, and how to compare states through an inner product that is intrinsic to the 1-geometry. The identification of 1-geometries involves the boundary condition that at spatial infinity the states should always be indistinguishable, which is a physical constraint corresponding to a preferred role for a strictly classical observer at infinity. The inner product is constructed through a mode decomposition in terms of the proper distance along the 1-geometry and is in this sense unique. The essential feature of an S-surface that leads to the difference in the states is that there is a large shift in the location of that surface in adjacent spacetimes. In terms of Kruskal coordinates near the horizon the shift is of order e^M in Planck units, a huge number, for a Planck scale difference in mass between the spacetimes. This shift is macroscopic and is more than sufficient to ensure that two states that began life at past null infinity with an overlap of order unity,

$$\langle \psi_{\mathcal{M}} \left[f(s)|\phi_{\mathcal{I}^-}(s) \right] \mid \psi_{\mathcal{M}+\delta\mathcal{M}} \left[f(s)|\phi_{\mathcal{I}^-}(s) \right] \rangle \sim 1$$

have virtually zero overlap on the S-surface:

$$\langle \psi_{\mathcal{M}} \left[f(s)|\phi_{\Sigma_S}(s) \right] \mid \psi_{\mathcal{M}+\delta\mathcal{M}} \left[f(s)|\phi_{\Sigma_S}(s) \right] \rangle \sim 0$$

Indeed even for a difference in masses of order e^{-M}, the overlap on an S-surface is close to zero. It is interesting to note that this scale leads to an entanglement entropy between the geometry and the matter on the S-surface of the same order as the usual black hole entropy.

We can deduce from these results that fluctuations in geometry prevent one from using quantum field theory on curved spacetime to determine the state of matter on an S-surface. Consequently, there is no way to compute both the Hawking radiation at future null infinity and the backreaction at the horizon. This strongly suggests that we should consider the entire solution to the Wheeler-DeWitt equation if we are to understand the physics of black holes, and not just its semi-classical projection.

Acknowledgements

References

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